ELEN 4810 Midterm Exam

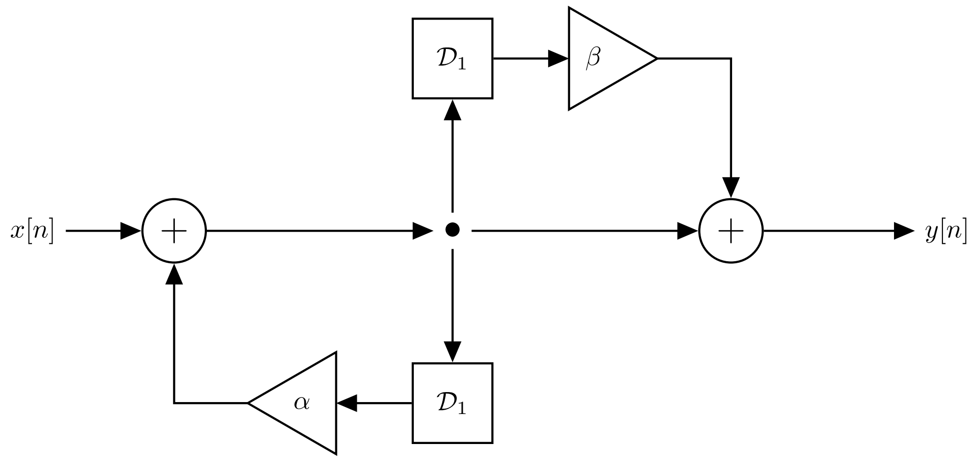
Monday, October 30, 2023, 1:10-3:10 PM. One sheet of handwritten notes is allowed. No electronics of any kind are allowed. Please record your answers in the exam booklet. Raise your hand if you need additional scratch paper.

There are a total of 4 questions. Good luck!

Name:

Uni:

1. Systems in Time and Frequency. Consider the causal linear, time invariant system corresponding to the following block diagram:



Here, $\mathcal{D}_1$ denotes an ideal delay by one sample, and the triangular blocks denote multiplication by complex scalars α and β, respectively.

Please answer the following questions:

Part (a). For what α and β is the system *stable*?

[Note: for full credit, please specify all possible values of (α, β)]

Part (b). What is the *frequency response* $H(e^{j\omega})$ of the system?

[Note: here, you only need to consider values of (α, β) for which the system is stable]

Part (c). What is the *impulse response* $h[n]$ of the system? Is this an FIR or IIR system?

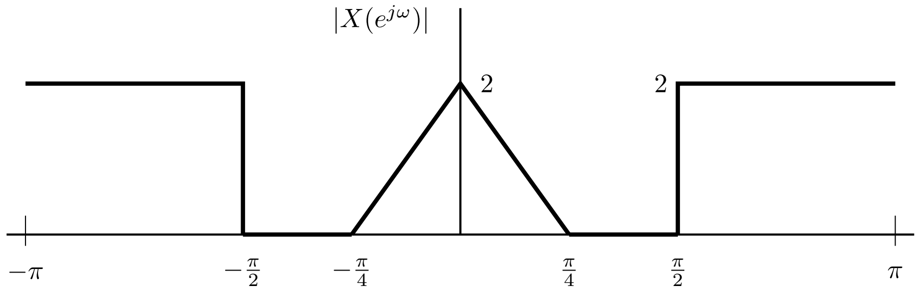
Part (d). Consider the constant input $x[n] = 1$. *For what choices of* (α, β) *is the output* $y[n] = 0$ *zero for all* n*?* Please make your answer as broad as possible, or if no such (α, β) exist, explain why.

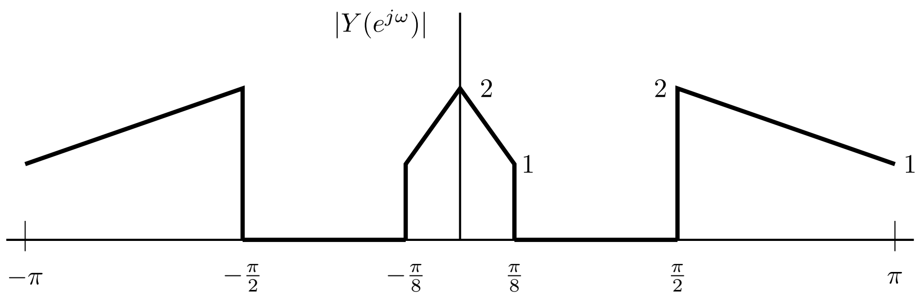
Part (e). Consider the input $x[n] = 1 + 3(-1)^n$. *For what choices of* (α, β) *is the output* $y[n] = 0$ *zero for all* n*?* Again, please make your answer as broad as possible, or if no such (α, β) exist, explain why.

Answer to Problem 1:

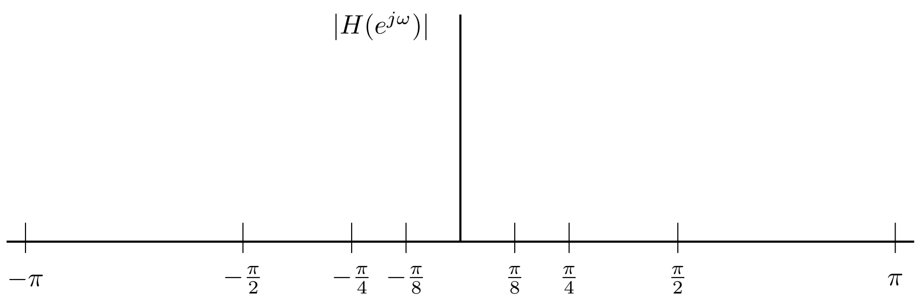
2. LTI Systems in Frequency Domain. A signal $x[n]$ is input to an LTI system with impulse response $h[n]$, producing output $y[n]$. Below, we plot magnitude and phase of $X(e^{j\omega})$ and $Y(e^{j\omega})$.

Part (a). *Please plot the magnitude and phase of $H(e^{j\omega})$. Please label your graph as clearly as possible (including heights), and indicate any points for the magnitude or phase response cannot be determined:*





Your answer to Part (a) here:

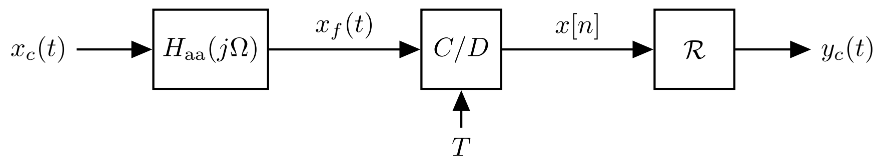


Part (b). For which of the following signals $x[n]$ is it possible to determine the impulse response $h[n]$ from the convolution $y[n] = x[n] * h[n]$? Please briefly explain your answers:

- 1. $x[n] = \frac{\sin\left((n-1)\pi/5\right)}{(n-1)\pi/5}$.
- 2. $x[n] = \cos(n\pi/4)$
- 3. $x[n] = \delta[n-10]$
- 4. $x[n] = (1/4)^n u[n]$

Answer to Problem 2:

3. Sampling and Aliasing. Consider the following block diagram:



Here,

- $x_c(t)$ is a continuous time signal
- $H_{\text{aa}}(j\Omega)$ is a continuous time filter whose frequency response will be specified below
- $x_f(t)$ is a continuous-time signal
- The box marked $\mathcal{R}$ will be specifed below.

Please answer the following questions:

Part (a). Suppose that $\mathcal{R}$ is an ideal discrete to continuous converter, with period T seconds. We wish to ensure that $y_c(t) = x_c(t)$ for the broadest possible set of inputs $x_c(t)$.

How should we choose $H_{\text{aa}}(j\Omega)$*?*

[Note: please specify H_{aa} as a function of Ω. Your answer should depend on T]

Part (b). *With your choice of* H_{aa} *from part (a), for which inputs* x_c *can we guarantee that* $y_c(t) = x_f(t)$*?*

[Note: Please make your answer as broad as possible for full credit. Please notice that Part (b) of the question asks whether $y_c = x_f$, *not* whether $y_c = x_c$!]

Part (c). Now suppose we are interested in reconstructing ***bandlimited*** signals $x_c(t)$ satisfying

$$X_c(j\Omega) = 0, \quad |\Omega| \geq \Omega_M. \tag{1}$$

Suppose further that we know that the input signal $x_c(t)$ is ***real valued***. We can use this property to reduce the sampling rate, by making different choices of the filter H_{aa} and the block $\mathcal{R}$.

Please specify a filter $H_{aa}(j\Omega)$*, a sampling period* T*, and describe a reconstruction procedure* $\mathcal{R}$ *which ensures* $y_c(t) = x_c(t)$ *for all such bandlimited, real-valued* $x_c(t)$*. For full credit, please make* T *as large as possible!*

[Hint: You may find it helpful to note that for a real valued signal $x_c(t)$, the Fourier transform is conjugate symmetric: $X_c(j\Omega) = X_c(-j\Omega)^*$]

Answer to Problem 3:

4. Discrete Fourier Transform and Convolution with a Filterbank. Consider real-valued discrete-time signal $x[n]$, which satisfies

$$\begin{cases} x[n] > 0 & n = 0, 1, \dots, 127, \\ x[n] = 0. & \text{else} \end{cases} \tag{2}$$

We are given a collection of filters $h_1[n], \dots, h_7[n]$ satisfying

$$h_i[n] = \begin{cases} h_i[n] = \cos\left(\frac{2\pi n}{2^i - 1}\right) & 0 \le n \le 2^i - 1. \\ 0 & \text{else} \end{cases} \tag{3}$$

We wish to compute the (linear) convolutions $y_1 = x * h_1, y_2 = x * h_2, \dots, y_7 = x * h_7$ **of** x **with each of the filters** h_i**.**

Please consider the following proposed procedure: we compute the discrete fourier transforms

$$\begin{aligned} X &= \mathrm{DFT}_N\{x\}, \\ H_i &= \mathrm{DFT}_N\{h_i\}, \qquad i = 1, \dots, 7. \end{aligned}$$

and set

$$\widehat{y}_i[n] \;=\; \mathrm{DFT}_N^{-1}\{H_i[k]X[k]\}, \qquad i = 1, \dots, 7.$$

Please answer the following questions about this procedure:

Part 1. Suppose we set $N = 150$. *Which of the convolutions is computed correctly?* I.e., for which values of i is $\widehat{y}_i[n] = y_i[n]$ for all n?

Part 2. *What is the smallest N for which* **all** *of the convolutions are computed correctly?* I.e., for which N is $\widehat{y}_i[n] = y_i[n]$ for all n and all $i = 1, \dots, 7$?

Part 3. Suppose again that $N = 150$, but now that we only care about values of n satisfying $10 \le n \le 118$. For which i is it true that

$$\widehat{y}_i[n] = y_i[n] \quad n = 10, \dots, 119 \quad ? \tag{4}$$

Answer to Problem 4:

Scratch paper: